647510 **BSCabsCC6020** **Seat No :________**

B.Sc. Semester - 6 (*CBCS*) Examination

Oct/Nov -2020

MATHEMATICAL ANALYSIS-2 AND ABSTRACT ALGEBRA -2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

QUE:1 (A) Answer any two out of three. **[10]**

(1) X is metric space and E_1 and E_2 be two connected subsets of X and $E_1 \bigcap E_2 \neq \phi$ then show that $E_1 \bigcup E_2$ is connected.

(2) State and prove theorem of nested interval.

(3) State and prove Heine-Borel theorem.

(B) Answer any one out of two. **[4]**

(1) Show that set of all integers is countable set.

(2) Check whether the following subsets of R are compact.

(i) $\{2,4,6,8,\ldots\ldots\ldots\}$ (ii) $(2,4) \bigcap (4,5)$

(iii) $\{\frac{1}{n} / n \in N\} \bigcup \{0\}$ (iv) N

QUE:2 (A) Answer any two out of three. **[10]**

(1) if f(t) is continuous for all $t \geq 0$ and f'(t) is piecewise continuous then prove that L{f'(t)} Exists, provided $t \xrightarrow{\lim} \infty$ $[e^{-st}f(t)]=0$ and $L\{f'(t)\}=sL\{f(t)\}-f(0)=s\overline{f}(s)-f(0)$. Also find $L[\sin^3(2t)]$.

(2) State and prove change the scale property. Using this property find L[sinh(at)sin(at)].

(3) Find : L[tsin(at)] using derivative of laplace transform.

(B) Answer any one out of two. **[4]**

(1) Find : $L^{-1}[\frac{1}{(s^2+a^2)^2}]$

(2) Find : $L^{-1}[\frac{3s+5}{(s+1)^4}]$

QUE:3 (A) Answer any two out of three. **[10]**

(1) Using convolution theorem Find $L^{-1}[\frac{1}{s\ (s^2+4\ )}]$.

(2) Prove that if $L\{f(t)\}=\bar{f}(s)$ then $L[\int_0^t f(u)du]=\frac{1}{s}\bar{f}(s)$. Using this result Find $L^{-1}[\frac{1}{s^3(s^2+a^2)}]$

(3) If $L\{f(t)\}=\bar{f}(s)$ and $\frac{f(t)}{t}$ has laplace transform then prove

That $L\{\frac{f(t)}{t}\}=\int_s^\infty \bar{f}(s)ds$. Using this result Find $L[\frac{\sin(t)}{t}]$.

(B) Answer any one out of two. **[4]**

(1) By using integration of laplace transform find $L[e^{-at}-e^{-bt}]$.

(2) Find: $L[e^{-4t}\int_0^t t\sin(3t)dt]$.

QUE:4 (A) Answer any two out of three. **[10]**

(1) $(G,*)$ and (G',Δ) be two groups. $\phi : G \rightarrow G'$ is homomorphism. Show that if K is normal subgroup of $\phi(G)$ then $\phi^{-1}(K)$ is normal subgroup of G.

(2) Show that every finite integral domain is field.

(3) Prove that characteristic of integral domain is either Zero or prime number.

(B) Answer any one out of two. **[4]**

(1) For non zero polynomial $f(x),g(x) \in D[X]$ show that $f(x)g(x)]=[f(x)]+[g(x)]$.

(2) Illustrate with example of subring of ring which is right ideal but not left ideal.

QUE:5 (A) Answer any two out of three. **[10]**

(1) State and prove division algorithm theorem of polynomial.

(2) Show that factors of $x^4+3x^3+2x+4=0$ in $Z_5[X]$ are $(x-1)^3(x+1)$.

(3) Show that $f(x)=x^4-2x^2+8x+1$ is irreducible over Q.

(B) Answer any one out of two. **[4]**

(1) Find G.C.D. d(x) of $f(x),g(x) \in Z_5[X]$. where $f(x)=x^3+2x^2+3x+2$ and $g(x)=x^2+4$. also express d(x) as $a(x)f(x)+b(x)g(x)$.

(2) Find: $[(2+3i)(4j+5k)^{-1}]^{-1}$

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